

Multiple Membership Feedback Models for Joint Longitudinal Responses

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A staged Bayesian framework for longitudinal health data

Workshop Objectives

PART I • CLINICAL PROBLEM AND RESEARCH QUESTIONS

- Recognize feedback in longitudinal health studies
- Distinguish external covariates from feedback-sensitive covariates
- Model a binary response and a continuous response jointly
- Represent patients receiving care from multiple clinical units
- Understand how Bayesian methods combine all parts of the model
- Interpret results from a simulated ICU dataset

Workshop Roadmap

Part I Clinical problem and research questions

Part III Construction of the joint feedback model

Part V Worked patient-level calculations

Part VII Clinical interpretation and application

Part II Simulated ICU dataset

Part IV Bayesian formulation and computation

Part VI Simulation results

Part VIII Conclusions and references

Motivating ICU Case Study

SIMULATED DATA • NOT REAL PATIENT DATA

- 1,000 simulated adult ICU patients
- Six consecutive six-hour observation windows per patient
- 6,000 patient-window observations
- Binary outcome: clinical deterioration requiring rescue care
- Continuous outcome: mean arterial pressure
- Treatment intensity changes during the ICU stay
- Patients may receive care from more than one ICU unit

Dynamic Clinical Care Process

During each observation window

- Clinical deterioration
- Mean arterial pressure
- Treatment and monitoring intensity
- ICU-unit participation

Before the next window

- Clinicians review the current outcomes
- Treatment intensity may change
- Updated treatment predicts the next outcomes

Clinical outcomes and treatment decisions evolve together over time.

Why Standard Longitudinal Models Struggle

IGNORING FEEDBACK CAN DISTORT TREATMENT-EFFECT ESTIMATES

- Treatment intensity is often treated as an external predictor
- Treatment may actually change in response to earlier outcomes
- Binary and continuous outcomes may be correlated
- Outcomes and treatment may persist over time
- Patients may receive care from several ICU units
- A single unit assignment ignores overlapping clinical responsibility

Main concern

Ignoring feedback may produce misleading treatment-effect estimates because treatment is partly determined by the patient's previous condition.

Research Questions

Feedback

1. Does deterioration at time t affect treatment before time $t+1$?
2. Does MAP at time t affect treatment before time $t+1$?

Subsequent outcomes

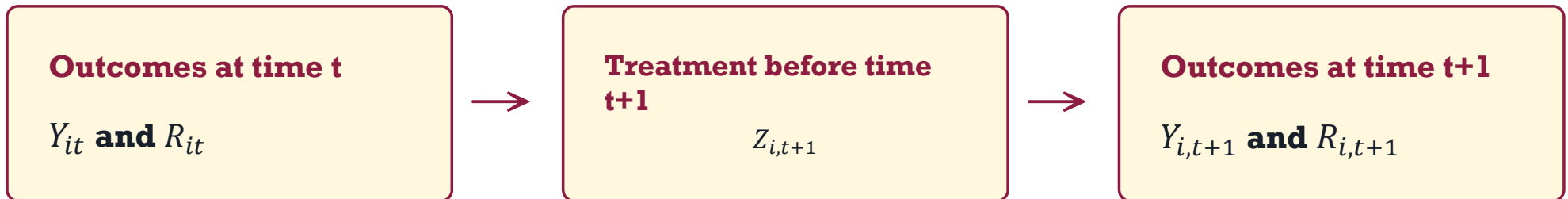
3. Does updated treatment predict later deterioration?
4. Does updated treatment predict later MAP?

Dependence and membership

5. How much persistence exists in the process?
6. Do overlapping ICU units explain residual variation?
7. What is lost when feedback or membership is omitted?

Timing of the Feedback Process

THE FUNDAMENTAL TEMPORAL ORDER



**The outcomes observed at time t inform treatment before the next window.
The updated treatment then predicts the outcomes at time t+1.**

During observation window t

The model observes:

- Binary deterioration status, Y_{it}
- Mean arterial pressure, R_{it}
- Current treatment intensity, Z_{it}

Between windows t and t+1

The outcomes observed during window t may change treatment intensity:

$$(Y_{it}, R_{it}) \rightarrow Z_{i,t+1}$$

During observation window t+1

The updated treatment intensity predicts the next outcomes:

$$Z_{i,t+1} \rightarrow (Y_{i,t+1}, R_{i,t+1})$$

Complete feedback sequence

$$(Y_{it}, R_{it}) \rightarrow Z_{i,t+1} \rightarrow (Y_{i,t+1}, R_{i,t+1})$$

Key message

The outcomes at time t do not change treatment at that same instant. They inform the treatment decision made before the next observation window begins.

Part II

Simulated ICU dataset

Simulation Disclosure and Purpose

SIMULATED DATA • NO REAL PATIENT RECORDS

Disclosure

- Every patient and measurement was simulated
- No protected health information is included
- The records do not represent Taichung Veterans General Hospital or MIMIC-IV
- The temporal relationships resemble a plausible ICU process

Purpose

- Demonstrate feedback between outcomes and treatment
- Represent binary and continuous outcomes jointly
- Include overlapping ICU-unit membership
- Evaluate recovery of known parameters
- Provide reproducible workshop calculations

Structure of the Simulated Dataset

ONE ROW REPRESENTS ONE PATIENT DURING ONE SIX-HOUR WINDOW

Dataset characteristic	Value
Patients	1,000
Windows per patient	6
Length of each window	6 hours
Patient-window observations	6,000
ICU units	30
Overlapping unit observations	2,679
Overlapping membership	44.7%

The number of patient-window observations is $1,000 \times 6 = 6,000$

Variables in the Simulated Dataset

Role	Notation	Variable	Interpretation
Binary response	Y_{it}	Deterioration	1 if rescue care was required
Continuous response	R_{it}	Mean arterial pressure	Average MAP during the window
Feedback covariate	Z_{it}	Treatment intensity	Vasopressor and monitoring intensity
Lagged responses	$Y_{i,t-1}; R_{i,t-1}$	Prior outcomes	Status during the preceding window
Lagged treatment	$Z_{i,t-1}$	Prior treatment	Treatment during the preceding window
External covariates	x_{it}	Patient characteristics	Age, sex, comorbidity, and acuity
Membership weights	ω_{itj}	ICU-unit exposure	Proportion of time in unit j

Important distinction

The baseline patient characteristics are treated as external covariates. Treatment intensity is feedback-sensitive because it may change in response to earlier outcomes.

How the Data Were Generated

1. Baseline patient characteristics

- Age, sex, comorbidity burden, and acuity
- Patient-specific latent effects

2. ICU-unit membership

- Primary unit with possible second-unit exposure
- Time-based weights sum to one within each window

$$\sum_j \omega_{itj} = 1$$

3. Longitudinal process

- Generate treatment, deterioration, and MAP
- Current outcomes update treatment before the next window

A Few Rows of the Simulated Data

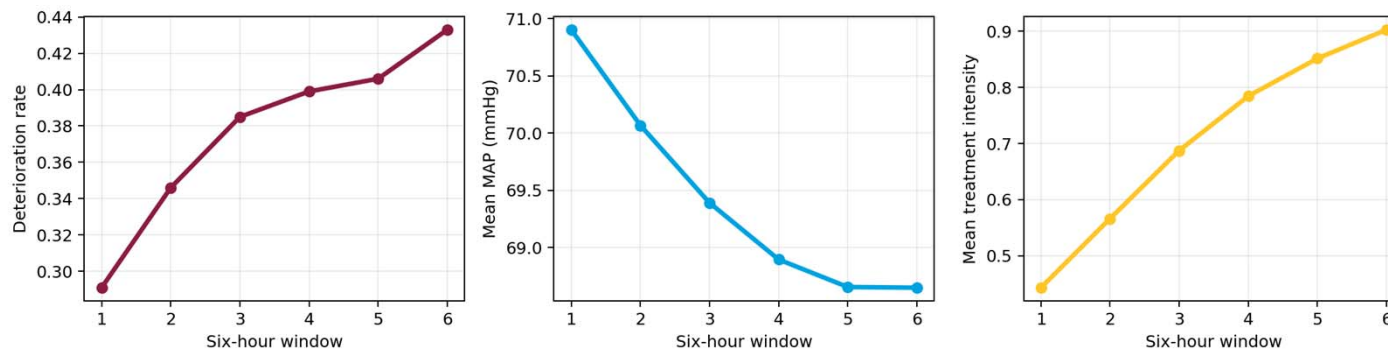
SIMULATED DATA • NOT REAL PATIENT RECORDS

Patient	Window	Y	MAP	Z	ICU-unit membership
1	1	1	73.25	0.16	Unit 4: 1.00
1	2	0	71.45	0.92	Unit 4: .70; Unit 3: .30
1	3	1	70.74	1.05	Unit 4: 1.00
1	4	0	69.37	0.91	Unit 4: 1.00
1	5	1	65.95	0.66	Unit 4: 1.00
1	6	0	61.09	1.25	Unit 4: 1.00
2	1	0	72.42	0.71	Unit 9: .74; Unit 24: .26
2	2	0	71.05	0.30	Unit 9: .58; Unit 24: .42

Descriptive Results from the Simulated Cohort

1,000 PATIENTS • 6,000 PATIENT-WINDOW OBSERVATIONS

Observed trajectories in the simulated ICU cohort



Interpretation

Across the six simulated windows:

- The deterioration rate increases. Mean arterial pressure decreases. Treatment intensity increases.

These descriptive patterns are consistent with a dynamic process in which worsening patient status leads to greater treatment intensity. However, the descriptive results alone cannot separate feedback, persistence, patient heterogeneity, and ICU-unit effects. The joint model is required for that purpose.

Overall summary

- Deterioration rate: 37.7%
- Mean MAP: 69.43 mmHg
- MAP SD: 4.31 mmHg
- Mean treatment: 0.706
- Overlapping membership: 44.7%

Worsening status coincides with increasing treatment intensity.

Part III

Building the model

Notation

$$R_{it}^* = (R_{it} - 70) / 8$$

Notation	Meaning
Y_{it}	Binary deterioration indicator
R_{it}	Mean arterial pressure
R_{it}^*	Standardized mean arterial pressure
Z_{it}	Treatment and monitoring intensity
x_{it}	External patient and clinical covariates
b_{iY}, b_{iR}, u_{iZ}	Patient-specific effects
w_{itj}	Proportion of the window spent in ICU unit j
a_{jY}, a_{jR}, a_{jZ}	ICU-unit effects

Standardized $R_{it}^* = (R_{it} - 70) / 8 = 0.80$

=means that the patient's MAP is 0.80 standard deviations below the reference mean of 70 mmHg.

Component 1: Binary Deterioration Outcome

BINARY RESPONSE

- $Y_{it} = 1$ when the patient requires rescue care during window t
- $Y_{it} = 0$ when rescue care is not required
- The model estimates $\Pr(Y_{it} = 1)$
- Predictors include current treatment, prior deterioration, patient characteristics, patient susceptibility, and ICU-unit membership
- Clinical question: Does current treatment help predict deterioration after accounting for prior condition and care setting?

Binary-Response Model

COMPONENT 1

$$\text{logit}\{\Pr(Y_{it} = 1)\} = \beta_{0Y} + \mathbf{x}_{it}'\boldsymbol{\beta}_Y + \gamma_Y Z_{it} + \rho_Y Y_{i,t-1} + b_{iY} + m_{itY}$$

$$m_{itY} = \sum_j w_{itj} a_{jY}$$

- γ_Y : treatment-intensity association
- ρ_Y : persistence in deterioration
- b_{iY} : patient-specific contribution
- m_{itY} : weighted ICU-unit contribution

Interpretation of the Binary Model

ASSOCIATION IS NOT AUTOMATICALLY CAUSATION

- γ_Y is the change in log odds associated with a one-unit increase in treatment intensity
- $\exp(\gamma_Y)$ is the corresponding odds ratio
- ρ_Y compares patients who did and did not deteriorate during the preceding window
- A positive γ_Y does not prove that treatment causes deterioration
- Patients in poorer condition may receive more intensive treatment; the feedback model represents this selection process

Important caution

A positive value of γ_Y does not necessarily mean that treatment causes deterioration. Patients with more severe conditions may receive more intensive treatment.

The feedback model helps represent this treatment-selection process.

Component 2: Continuous MAP Outcome

CONTINUOUS RESPONSE

- R_{it} is mean arterial pressure during observation window t
- Current treatment intensity may predict MAP
- Prior MAP represents physiological persistence
- Prior deterioration may affect current MAP
- Patient and ICU-unit effects represent additional heterogeneity
- Clinical question: How is treatment intensity associated with subsequent blood pressure after accounting for prior MAP and patient heterogeneity?

Continuous-Outcome Model

COMPONENT 2

$$R_{it} = \beta_{0R} + x'_{it}\beta_R + \gamma_R Z_{it} + \rho_R R_{it-1}^* + \delta_R Y_{i,t-1} + b_{iR} + m_{itR} + \varepsilon_{itR}$$

$$m_{itR} = \sum_j w_{itj} a_{jR} \text{ and } \varepsilon_{itR} \sim \mathcal{N}(0, \sigma_R^2)$$

γ_R : treatment
association

ρ_R : MAP
persistence

δ_R : contribution of prior
deterioration

Model components

- β_{0R} continuous-model intercept
- $x'_{it}\beta_R$: effects of external covariates
- γ_R : treatment-intensity association
- ρ_R MAP persistence
- δ_R : effect of prior deterioration on current MAP
- b_{iR} : patient-specific effect
- m_{itR} : multiple-membership contribution
- ε_{itR} : residual variation

Interpretation of the Continuous Model

CONTINUOUS-OUTCOME PARAMETERS

- γ_R is the expected MAP difference associated with a one-unit increase in treatment intensity
- ρ_R measures how prior standardized MAP predicts current MAP
- δ_R measures the expected MAP difference associated with prior deterioration
- b_{iR} represents patient-specific physiology
- m_{itR} represents the weighted contribution of the ICU units providing care

Component 3: Feedback-Sensitive Treatment

FEEDBACK COVARIATE

- Treatment intensity is not treated as an entirely external predictor
- Deterioration during time t may change treatment before time $t+1$
- MAP during time t may change treatment before time $t+1$
- Treatment intensity may persist across windows
- Patients and ICU units may differ in their treatment tendencies
- Central timing: $(Y_{it}, R_{it}) \rightarrow Z_{i,t+1}$

The patient's current outcomes help determine the treatment used in the next window.

Feedback and Persistence Parameters

COMPONENT 3

Parameter	Question answered	Expected direction
ϕ_Z	Does treatment persist across windows?	Positive
λ_Y	Does deterioration increase later treatment?	Positive
λ_R	Does higher MAP change later treatment?	Negative
α_Z	How do external covariates affect treatment?	Covariate-specific
u_{iZ}	Does the patient have a treatment tendency?	Patient-specific
$m_{i,t+1,Z}$	Do the participating ICU units affect treatment?	Unit-specific

Feedback Model

THE FUNDAMENTAL TEMPORAL ORDER

$$Z_{i,t+1} = \alpha_0 + \phi_Z Z_{it} + \lambda_Y Y_{it} + \lambda_R R_{it}^* + x'_{it} \alpha_Z + u_{iZ} + m_{i,t+1,Z} + \varepsilon_{i,t+1,Z}$$

$$m_{i,t+1,Z} = \sum_j w_{i,t+1,j} a_{jZ}$$

$$\varepsilon_{i,t+1,Z} \sim \mathcal{N}(0, \sigma_Z^2),$$

ϕ_Z represents treatment persistence

λ_Y and λ_R are the feedback parameters

Current outcomes affect treatment before the next window

$$(Y_{it}, R_{it}, Z_{it}) \rightarrow Z_{i,t+1} \rightarrow (Y_{i,t+1}, R_{i,t+1})$$

Component 4: Multiple Membership

OVERLAPPING CLINICAL RESPONSIBILITY

- A patient may receive care from more than one ICU unit during a six-hour window
- A single-unit assignment discards part of the patient's care history
- Weights represent the proportion of the window spent in each unit
- Every weight lies between zero and one
- The weights for one patient-window sum to one

Multiple-Membership Contributions

COMPONENT 4

Binary response: $m_{itY} = \sum_j w_{itj} a_{jY}$

Continuous response: $m_{itR} = \sum_j w_{itj} a_{jR}$

Treatment intensity: $m_{itZ} = \sum_j w_{itj} a_{jZ}$

- The same time-based weights may enter all three equations
- The ICU-unit effects can differ across deterioration, MAP, and treatment
- $0 \leq \sum_j w_{itj} = 1$

Worked Multiple-Membership Example

FOUR HOURS IN UNIT A • TWO HOURS IN UNIT B

$$w_A = 4/6 \text{ and } w_B = 2/6$$

$$m_{itY} = 0.67(0.30) + 0.33(-0.15)$$

$$m_{itY} = 0.1515 \approx 0.15$$

- The patient spent four hours in Unit A and two hours in Unit B
- Unit A effect: 0.30; Unit B effect: -0.15
- The model assigns the weighted contribution 0.15

Complete Joint Feedback Model

THREE LINKED PROCESSES

Binary deterioration

$$\text{logit}\{\Pr(Y_{it} = 1)\} = \beta_{0Y} + x'_{it}\beta_Y + \gamma_Y Z_{it} + \rho_Y Y_{i,t-1} + b_{iY} + m_{itY}$$

Continuous MAP

$$R_{it} = \beta_{0R} + x'_{it}\beta_R + \gamma_R Z_{it} + \rho_R R_{it-1}^* + \delta_R Y_{i,t-1} + b_{iR} + m_{itR} + \varepsilon_{itR}$$

Treatment feedback

$$Z_{i,t+1} = \alpha_0 + \phi_Z Z_{it} + \lambda_Y Y_{it} + \lambda_R R_{it}^* + x'_{it}\alpha_Z + u_{iZ} + m_{i,t+1,Z} + \varepsilon_{i,t+1,Z}$$

Main contribution

The model represents outcomes, treatment decisions, feedback, persistence, patient heterogeneity, and overlapping ICU-unit membership within one framework.

Parameters and Scientific Questions

Parameter	Scientific interpretation
λ_Y	Treatment intensity and deterioration
λ_Y	Treatment intensity and MAP
ρ_Y	Persistence in deterioration
ρ_R	Persistence in MAP
δ_R	Prior deterioration and current MAP
ϕ_Z	Persistence in treatment intensity
λ_Y	Feedback from deterioration to later treatment
λ_R	Feedback from MAP to later treatment
$b_{iY} b_{iR}, , u_{iZ}$	Patient-specific tendencies
a_{jY}, a_{jR}, a_{jZ}	ICU-unit effects on the three processes

How the Model Creates Dependence

Feedback

$$(Y_{it}, R_{it}) \rightarrow Z_{i,t+1}$$

Treatment effects

$$Z_{it} \rightarrow (Y_{it}, R_{it})$$

Serial persistence

Prior Y, R, and Z predict their later values-

$$(Y_{i,t-1}) \rightarrow Y_{i,t}$$

$$(R_{i,t-1}) \rightarrow R_{i,t}, (Z_{i,t-1}) \rightarrow Z_{i,t}$$

Shared latent effects

Patient and ICU-unit effects may be correlated

After conditioning on covariates, treatment, patient effects, and ICU-unit effects, the remaining outcome components are assumed conditionally independent.

Part IV

Bayesian formulation and computation

Why Use a Bayesian Model?

ONE MODEL FOR ALL SOURCES OF UNCERTAINTY

- Estimates all connected components jointly
- Propagates uncertainty through outcomes, feedback, patients, and ICU units
- Provides probability distributions for every model parameter
- Accommodates correlated patient and ICU-unit effects
- Produces patient-specific and unit-specific predictions
- Supports posterior predictive model checking

A Bayesian model estimates all components jointly rather than treating estimated random effects or feedback processes as known.

Bayesian advantages

- Propagates uncertainty through the complete model; Provides probability distributions for all parameters; Accommodates correlated patient and ICU-unit effects
- Produces patient-specific and unit-specific predictions, Supports posterior predictive model checking, Allows prior information when scientifically justified

Bayesian Hierarchy

Level 1: Observed outcomes

$$Y_{it} \sim \text{Bernoulli}(p_{it}) \quad R_{it} \sim N(\mu_{itR}, \sigma_R^2) \quad Z_{i,t+1} \sim N(\mu_{i,t+1,Z}, \sigma_Z^2)$$

Level 2: Regression models

$$\text{logit}(p_{it}) = \eta_i \quad \mathbf{E}(R_{it}) = \mu_{itR} \quad \mathbf{E}(Z_{i,t+1}) = \mu_{i,t+1,Z}$$

Level 3: Latent effects

Patient-specific effects and weighted ICU-unit effects

Level 4: Prior distributions

Priors for coefficients, standard deviations, and correlation matrices

Correlated Patient and ICU-Unit Effects

Patient-specific effects

$$b_{iY} = (b_{iY}, b_{iR}, u_{iZ})^T \sim \mathcal{N}_3(0, \Sigma_b)$$

ICU-unit effects

$$a_j = (a_{jY}, a_{jR}, a_{jZ})^T \sim \mathcal{N}(0, \Sigma_a)$$

Σ_b describes

- Variation among patients
- Association among patient tendencies

Σ_a describes

- Variation among ICU units
- Association among unit tendencies

Off-diagonal covariance terms connect deterioration, MAP, and treatment after covariate adjustment

Prior Distributions

ILLUSTRATIVE WEAKLY INFORMATIVE PRIORS

Regression coefficients

$$\beta_k \sim \text{Normal}(0, 2.5^2)$$

Intercepts

$$\beta_{0Y} \beta_{0R}, \alpha_0 \sim \text{Normal}(0, 5^2)$$

Standard deviations

$$\sigma \sim \text{HalfNormal}(0, 2.5)$$

Patient correlation matrix

$$\Omega_b \sim \text{LKJ}(2)$$

ICU-unit correlation matrix

$$\Omega_a \sim \text{LKJ}(2)$$

The final analysis should examine sensitivity to reasonable alternative priors.

Joint Likelihood

$\mathbf{D} = \{Y_{it}, R_{it}, Z_{it}x_{it}, w_{it},\}$ denote the observed longitudinal data.

Patient-window contribution = $p(Y_{it} | p_{it}) \times p(R_{it} | \mu_{itR}, \sigma_R^2) \times p(Z_{i,t+1} | \mu_{i,t+1,Z}, \sigma_Z^2)$

$$\mathcal{L}((\Theta; \mathcal{D})) = \prod_{i=1}^N \prod_{t=1}^{T_i-1} p(Y_{it}|p_{it})p(R_{it}|\mu_{itR}, \sigma_R^2)p(Z_{i,t+1}|\mu_{i,t+1,Z}, \sigma_Z^2)$$

- The last window contributes outcomes but no subsequent treatment observation
- Initial values Y_{i1}, R_{i1}, Z_{i1} are conditioned upon or modeled separately

Posterior Distribution

Θ contains coefficients, feedback parameters, persistence parameters, latent effects, variances, and correlations.

$$p(\Theta \mid \mathbf{D}) = p(\mathbf{D} \mid \Theta)p(\Theta) / p(\mathbf{D})$$

$$p(\Theta \mid \mathbf{D}) \propto p(\mathbf{D} \mid \Theta)p(\Theta)$$

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}$$

Posterior Computation and Diagnostics

Computation

- Hamiltonian Monte Carlo with NUTS
- Four independent chains
- 2,000 warm-up iterations per chain
- 2,000 retained iterations per chain
- 8,000 retained posterior draws

Diagnostics

- R-hat below 1.01
- Adequate effective sample sizes
- Trace plots and Monte Carlo error
- Divergences, energy, and tree depth
- Posterior predictive checks

Convergence shows that sampling worked. Posterior predictive checks assess whether the model reproduces important features of the data.

Part V

Worked patient computation

Following one simulated patient from window 3 to window 4

Treatment-Feedback Calculation

WINDOW 3 OUTCOMES DETERMINE TREATMENT BEFORE WINDOW 4

$$E(Z_{i,4}) = \alpha_0 + \phi_Z Z_{i3} + \lambda_Y Y_{i3} + \lambda_R R_{i3}^* + u_{iZ}$$

Patient and parameter values

$$\alpha_0 = 0.20$$

$$\phi_Z = 0.55$$

$$\lambda_Y = 0.50$$

$$\lambda_R = -0.20$$

$$Z_{i3} = 0.60$$

$$Y_{i3} = 1$$

$$R_{i3}^* = -0.80$$

$$u_{iZ} = 0.10$$

$$E(Z_{i,4}) = 0.20 + 0.55(0.60) + 0.50(1) - 0.20(-0.80) + 0.10$$

$$E(Z_{i,4}) = 0.20 + 0.33 + 0.50 + 0.16 + 0.10$$

$$E(Z_{i,4}) = 1.29$$

Deterioration and low MAP increase treatment intensity before window 4 begins.

Multiple-Membership Calculation

FOUR HOURS IN UNIT A • TWO HOURS IN UNIT B

$$w_A = \frac{4}{6} = 0.67 \quad w_B = 2/6 = 0.33$$

Binary-outcome contribution

$$m_{i4Y} = 0.67(0.30) + 0.33(-0.15)$$

$$m_{i4Y} = .15$$

Continuous-outcome contribution

$$m_{i4R} (30.0 -)33.0 + (60.0)67.0 =$$

$$m_{i4R} = .30$$

The same membership weights produce different contributions because the unit effects differ across outcomes.

Binary-Outcome Calculation

PREDICTED DETERIORATION DURING WINDOW 4

$$\text{logit}\{\text{Pr}(Y_{i,4} = 1)\} = \beta_{0Y} + \gamma_Y Z_{i4} + \rho_Y Y_{i,3} + \mathbf{m_i4Y} + b_{iY} + m_{i4Y}$$

$$\eta_{\mathbf{i4Y}} = -1.20 + 0.80(1.29) + 0.35(1) + 0.15$$

$$\eta_{\mathbf{i4Y}} = -1.20 + 1.032 + 0.35 + 0.15 = 0.332$$

$$\text{Pr}(Y_{i,4} = 1) = \exp(0.332) / \{1 + \exp(0.332)\}$$

$$\text{Pr}(Y_{i,4} = 1) = 0.582$$

The patient has a 58.2% predicted probability of deterioration during window 4.

Continuous-Outcome Calculation

PREDICTED MEAN ARTERIAL PRESSURE DURING WINDOW 4

$$E(R_{i4}) = \beta_{0R} + \gamma_R Z_{i4} + \rho_R R_{i3*} + \delta_R Y_{i3} + b_{iR} + m_{i4R}$$

Values used

$\beta_{0R} = 72.0$
 $\gamma_R = -3.0$
 $\rho_R = 2.5$
 $\delta_R = -1.0$
 $Z_{i4} = 1.29$
 $R_{i3*} = -0.80$
 $Y_{i3} = 1$
 $b_{iR} = 0.50$
 $m_{i4R} = 0.30$

$$E(R_{i4}) = 72.0 - 3.0(1.29) + 2.5(-0.80) - 1.0(1) + 0.50 + 0.30$$

$$E(R_{i4}) = 72.0 - 3.87 - 2.00 - 1.00 + 0.50 + 0.30$$

$$E(R_{i4}) = 65.93 \text{ mmHg}$$

This is the conditional expected MAP; the observed value may differ because of residual variation.

Complete Patient Update

THE FULL TRANSITION FROM WINDOW 3 TO WINDOW 4

Window 3: observed status

$Y_{i,3} = 1$
 $R_{i,4} = 63.6 \text{ mmHg}$
 $R_{i3*} = -0.80$
 $Z_{i3} 60.0 =$



Treatment before window 4

$E(Z_{i,4}) 1.29 = ($
 $m_{i,4Y} = 0.15$
 $m_{i,4R} = 0.30$



Window 4: predictions

$\Pr(Y_{i,4}) 0.582 = (1 =$
 $E(R_{i,4}) 65.93 = (\text{mmHg}$

Current outcomes update treatment.

Updated treatment and ICU-unit membership predict both next-window outcomes.

Part VI

Simulation results

Parameter recovery, prediction, and model checking

Simulation Design

SIMULATED DATA • 1,000 PATIENTS

Design feature	Value
Patients	1,000
Windows per patient	6
Patient-window observations	6,000
Six-hour windows	6 per patient
ICU units	30
Overlapping membership	44.7%
Random seed	20260926

Known feedback values

ϕ_Z **0.55**

λ_Y **0.48**

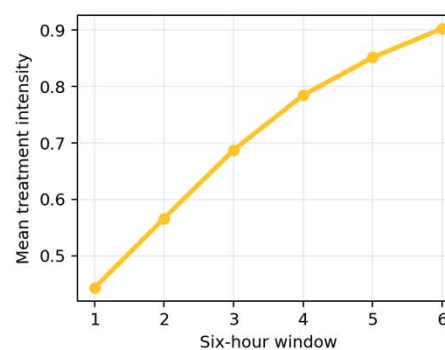
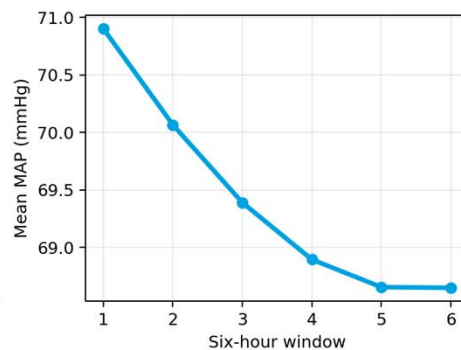
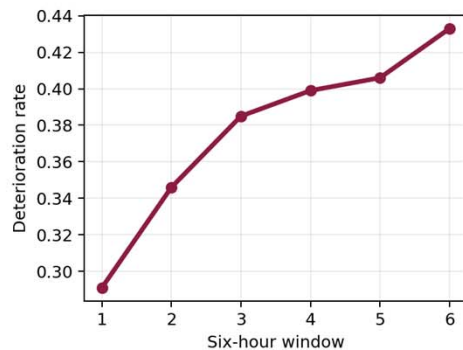
λ_R **-0.18**

Objective: recover the direction and approximate magnitude of treatment persistence and feedback.

Observed Trajectories in the Simulated Cohort

1,000 PATIENTS • SIX OBSERVATION WINDOWS

Observed trajectories in the simulated ICU cohort



Window 1 to Window 6

- Deterioration: 29.1% to 43.3%
- Mean MAP: 70.90 to 68.65
- Mean treatment: 0.443 to 0.903
- Descriptive trends motivate the feedback model

Recovery of the Feedback Parameters

ONE REPRODUCIBLE SIMULATED DATASET

Parameter	True	Estimate	Error	95% interval
ϕ_Z	0.550	0.545	-0.005	0.531 to 0.558
λ_Y	0.480	0.470	-0.010	0.460 to 0.479
λ_R	-0.180	-0.171	0.009	-0.181 to -0.160

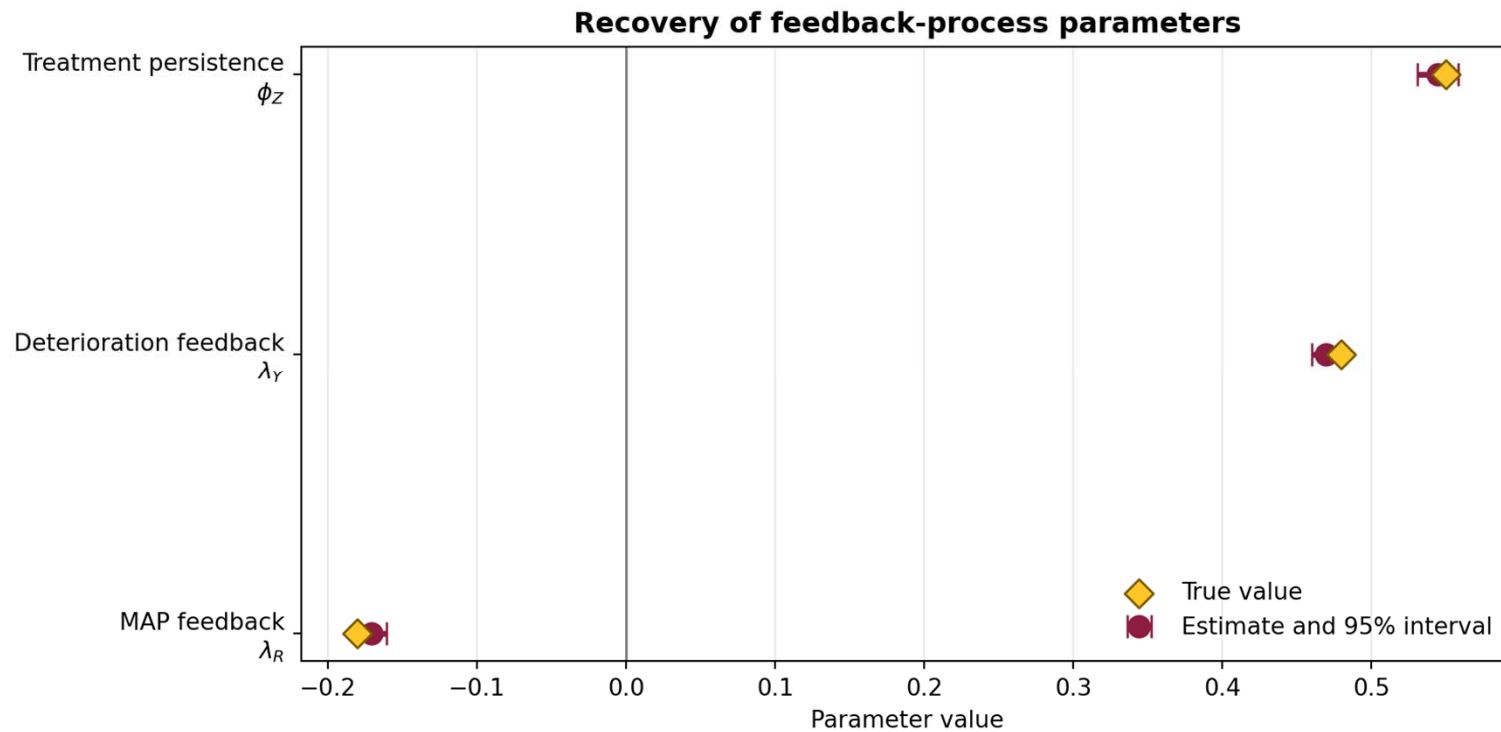
Treatment-model RMSE = 0.162

All three intervals exclude zero and retain the correct direction.

This is parameter recovery in one dataset, not a repeated-simulation coverage study.

Feedback Estimates and Uncertainty

TRUE VALUES AND FITTED 95% INTERVALS



The fitted model recovers the direction and approximate magnitude of all three feedback-process parameters.

Outcome-Model Estimates

ILLUSTRATIVE WORKSHOP FIT • SIMULATED DATA

Binary deterioration model

Parameter	Estimate	95% interval
γ_Y	0.866	0.670 to 1.063
ρ_Y	0.452	0.287 to 0.616

$$\exp(0.866) = 2.38$$

Adjusted association, not an automatic causal effect

Continuous MAP model

Parameter	Estimate	95% interval
γ_R	-2.805	-3.198 to -2.413
ρ_R	2.123	1.884 to 2.363

Higher treatment intensity is associated with lower MAP after adjustment.

Treatment responds to patient condition; therefore, treatment coefficients require careful interpretation.

Full Model Versus Reduced Models

LOWER PREDICTION ERROR IS BETTER

Deterioration prediction

Model	Brier score
Reduced: treatment omitted	0.219
Full: treatment included	0.216

Prediction error decreases by approximately 1.6%.

MAP prediction

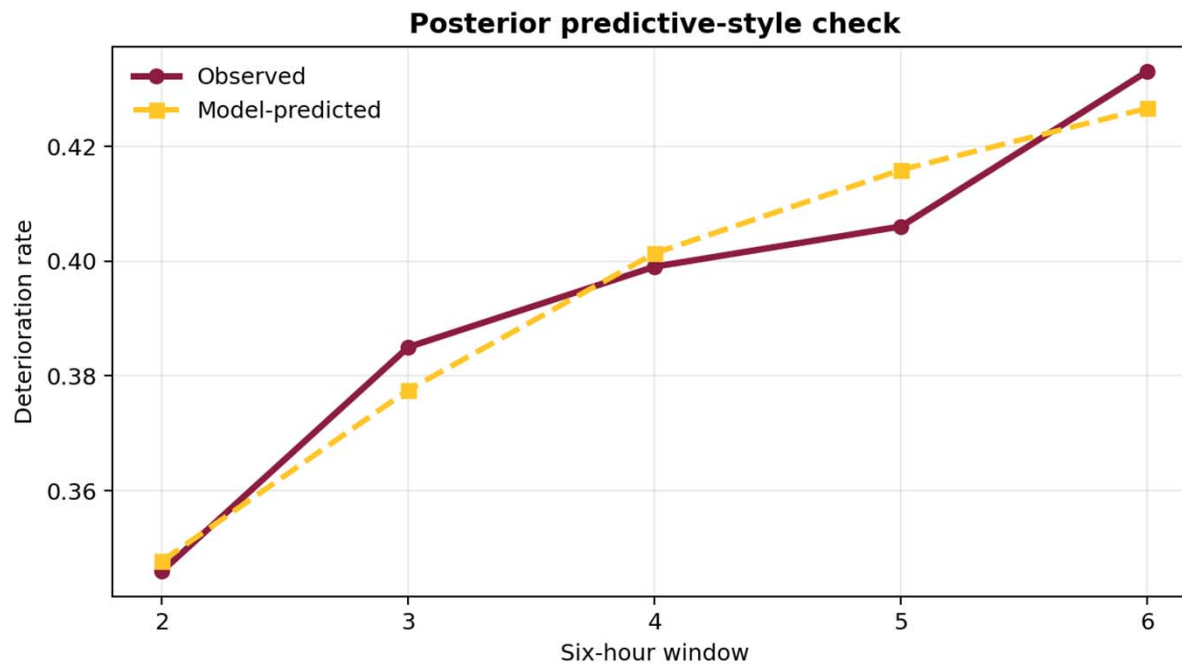
Model	RMSE
Reduced: treatment omitted	3.477
Full: treatment included	3.410

Prediction error decreases by approximately 1.9%.

Prediction improves modestly. The feedback equation remains necessary to represent how prior outcomes influence treatment.

Posterior Predictive and Model Checks

OBSERVED VERSUS MODEL-PREDICTED DETERIORATION



Additional checks

- Mean MAP by window
- Treatment trajectory
- Patient event-count distribution
- Within-patient persistence
- Residual MAP distribution
- ICU-unit variation
- Alternative weights and priors

The model reproduces the principal simulated patterns, but prediction does not establish causation.

Part VII

Interpretation and application

From statistical estimates to clinical understanding

What do the fitted feedback relationships mean—and how can the framework be used with real ICU data?

Interpreting the Simulated Results

WHAT THE FEEDBACK ESTIMATES TELL US

Treatment persistence

$$\phi_Z = 0.545$$

Treatment or monitoring intensity tends to persist. Intensive treatment in one window is likely to continue into the next.

Deterioration feedback

$$\lambda_Y = 0.470$$

A deterioration event at time t increases expected treatment intensity at time $t + 1$.

MAP feedback

$$\lambda_R = -0.171$$

Lower standardized MAP at time t leads to greater expected treatment intensity at time $t + 1$.

Clinical meaning: treatment is part of the evolving outcome process because clinicians respond to the patient's changing condition.

Caution: these are simulated-data results—not estimates from actual patients and not automatic causal effects.

Applying the Framework to Real ICU Data

POTENTIAL APPLICATION AT TAICHUNG VETERANS GENERAL HOSPITAL

Patient-window information needed

- Binary outcome: deterioration or adverse event
- Continuous outcome: MAP, oxygenation, or laboratory value
- Feedback-sensitive treatment or monitoring intensity
- Patient characteristics and time-varying predictors
- ICU-unit, ward, or clinical-team memberships

Questions the model can address

- How do previous outcomes alter subsequent treatment?
- How is treatment associated with later joint outcomes?
- How much variation occurs across patients and units?
- Does movement across units affect trajectories?
- What is lost when feedback or membership is omitted?

Application workflow



The framework treats treatment, outcomes, and clinical membership as one evolving longitudinal system.

Part VIII

Conclusions and references

Bringing the model components together

What Have We Learned?

THE PROBLEM, THE MODEL, AND THE CENTRAL MESSAGE

The clinical problem

- A binary clinical outcome
- A continuous clinical outcome
- Treatment changes in response to previous outcomes
- Patients may receive care from several units

The joint solution

- Outcome-to-treatment feedback
- Within-patient dependence
- Patient-specific heterogeneity
- Multiple clinical memberships
- Joint Bayesian uncertainty

Why it matters

- Treatment is part of the outcome process
- Changing care settings affect trajectories
- Separate models can miss key dependence
- Interpretation must respect clinical response

Treatment, outcomes, and clinical membership should be modeled as parts of one evolving system.

Main Conclusions

SIX LESSONS FROM THE WORKSHOP

- 1** Feedback changes the data-generating process.
- 3** Joint modeling preserves relationships between outcomes.
- 5** Bayesian modeling propagates uncertainty across components.

- 2** Treatment is not simply an external predictor.
- 4** Multiple membership represents changing care environments.
- 6** The simulated example demonstrates model feasibility.

Ignoring feedback or changing membership may oversimplify the clinical process and distort interpretation.

Limitations and Next Steps

MOVING FROM THE WORKSHOP SIMULATION TO REAL CLINICAL DATA

Current limitations

- Results are simulated—not estimates from real patients
- Clinical windows are prespecified
- Causal claims require additional assumptions
- Membership weights must be clinically meaningful
- Complex joint models can be computationally demanding

Next steps

- Apply the model to eICU, MIMIC-IV, or hospital data
- Compare alternative clinical time windows
- Study alternative membership weights
- Evaluate bias, coverage, and power across simulations
- Extend to nonlinear, delayed, or patient-specific feedback

The immediate goal is a clinically grounded application with transparent sensitivity analyses.

Final Perspective

MODEL THE CLINICAL PROCESS THAT GENERATED THE DATA



For researchers

- A unified framework for dependent outcomes, endogenous treatment, and changing membership

For clinicians

- A model reflecting that care decisions respond to condition and shape later trajectories

Model the evolving clinical system—not only the outcomes that were recorded.

Selected References

FOUNDATIONAL SOURCES USED IN THE WORKSHOP

Browne, W. J., Goldstein, H., & Rasbash, J. (2001). Multiple membership multiple classification models. *Statistical Modelling*, 1, 103–124.

Diggle, P. J., Heagerty, P., Liang, K.-Y., & Zeger, S. L. (2002). *Analysis of Longitudinal Data*. Oxford University Press.

Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., & Rubin, D. B. (2013). *Bayesian Data Analysis*. CRC Press.

Rigby, R. A., & Stasinopoulos, D. M. (2005). Generalized additive models for location, scale and shape. *Applied Statistics*, 54, 507–554.

Stasinopoulos, D. M., Rigby, R. A., Heller, G. Z., Voudouris, V., & De Bastiani, F. (2017). *Flexible Regression and Smoothing: Using GAMLSS in R*. CRC Press.

Stan Development Team. *Stan Modeling Language Users Guide and Reference Manual*.

Selby, Wilson, and Lu (2025): insert the complete citation exactly as listed in the paper.

Thank You and Discussion

Multiple Membership Feedback Models for Joint Longitudinal Responses

Questions for discussion

- Where does feedback arise in your longitudinal studies?
- Can patients belong to several clinical units or care teams?
- Which binary and continuous outcomes should be modeled jointly?
- How should time windows and membership weights be defined?

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